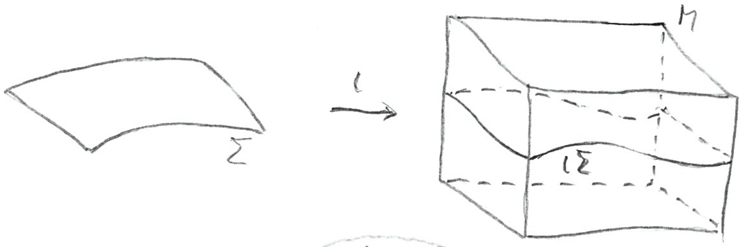
Podvarieta, tečné vektory a normálové formy

vnoření Σ do M

$\iota : \Sigma \to M$

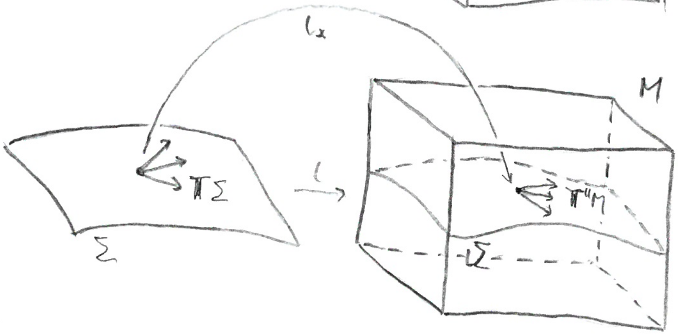


$\mathbb{T}\Sigma$ tečný bundl Σ

$\mathbb{T}^{\parallel}M \subset \mathbb{T}M$ tečné vektory Σ v $\mathbb{T}M$

$\iota_* : \mathbb{T}\Sigma \to \mathbb{T}^{\parallel}M \subset \mathbb{T}M$

$a \to \iota_* a$

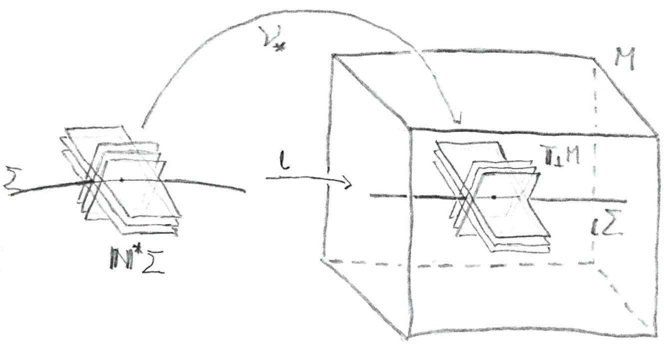


$\mathbb{N}^*\Sigma$ normálový bundl forem

$\mathbb{T}_{\perp}M \subset \mathbb{T}^*M$ normálové formy Σ v $\mathbb{T}^*M$

$\mathbb{T}_{\perp}M$ normálové formy anihil. $\mathbb{T}^{\parallel}M$

$k \in \mathbb{T}_{\perp}M \Leftrightarrow \forall a \in \mathbb{T}^{\parallel}M \quad k \cdot a = 0$



$\mathbb{N}^*\Sigma$ abstraktní vzor norm. forem $\mathbb{T}_{\perp}M$
prostor isomorfní s $\mathbb{T}_{\perp}M$ chápaný jako bundl nad Σ

$\nu_* : \mathbb{N}^*\Sigma \to \mathbb{T}_{\perp}M \subset \mathbb{T}^*M$

$k \to \nu_* k$

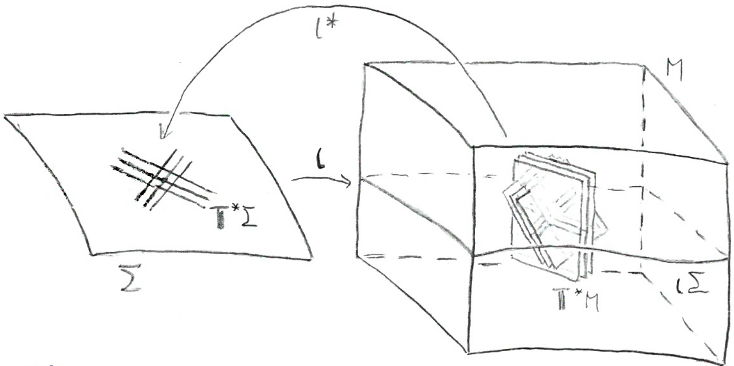
$\mathbb{T}^*\Sigma$ kotečný bundl Σ

$\iota^* : \mathbb{T}^*M \to \mathbb{T}^*\Sigma$ restrikce na $\mathbb{T}\Sigma$

$\omega \to \omega|_{\mathbb{T}\Sigma} = \iota^*\omega$

$\omega|_{\mathbb{T}\Sigma} \cdot a = \omega \cdot (\iota_* a)$

ignoruje norm. složku
nelze vnořit do $\mathbb{T}^*M$ $\quad \iota^*\mathbb{T}_{\perp}M = 0$



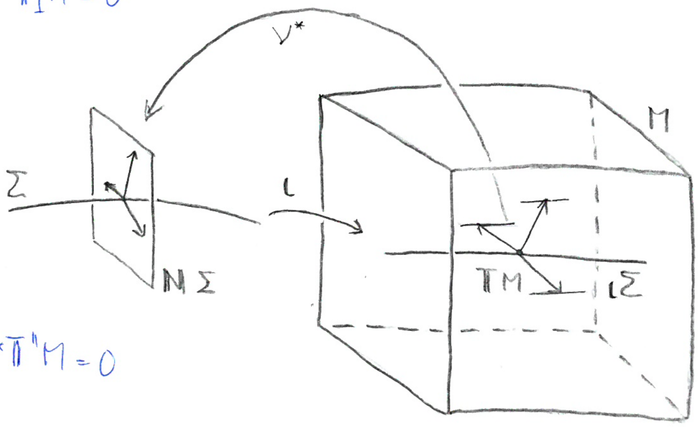
$\mathbb{N}\Sigma$ normálový bundl vektorů

$\nu^* : \mathbb{T}M \to \mathbb{N}\Sigma$ restrikce na $\mathbb{N}\Sigma$

$u \to u|_{\mathbb{N}\Sigma} = \nu^* u$

$k \cdot u|_{\mathbb{N}\Sigma} = (\nu_* k) \cdot u$

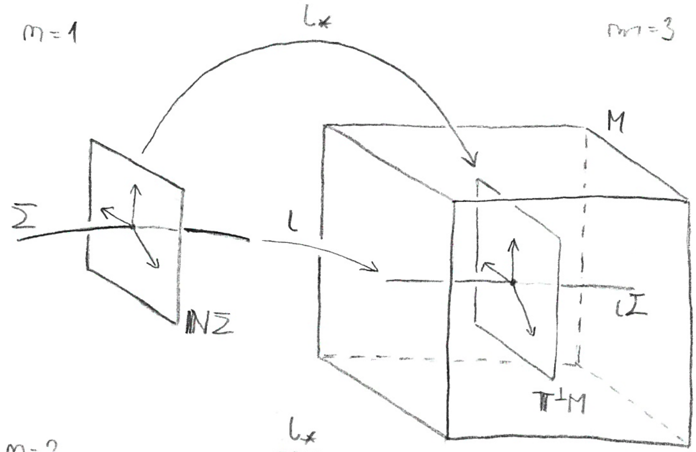
ignoruje tečnou složku
nelze vnořit do $\mathbb{T}M$ $\quad \nu^*\mathbb{T}^{\parallel}M = 0$

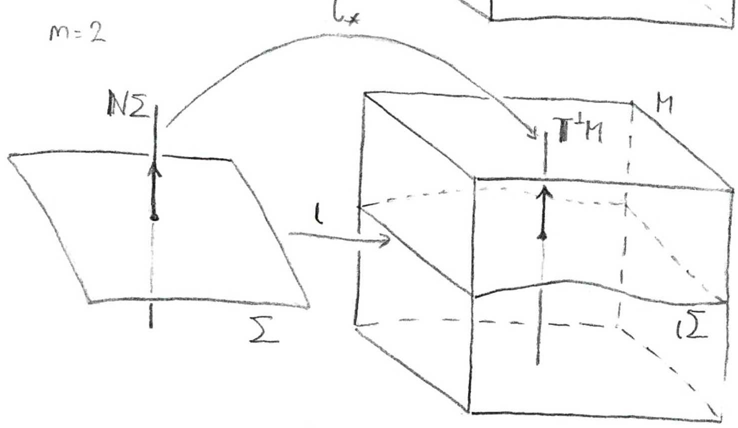


$\mathbb{T}^\perp M$ normálové vektory vnořené do $\mathbb{T}M$

$\iota_* : \mathbb{N}\Sigma \to \mathbb{T}^\perp M \subset \mathbb{T}M$

definováno pomocí $^{\perp}\delta$

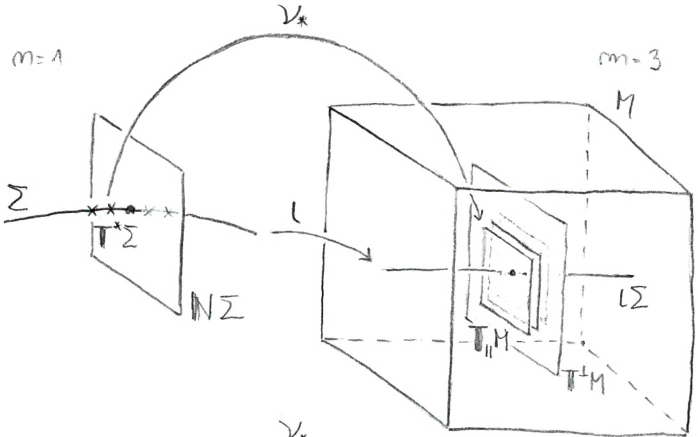


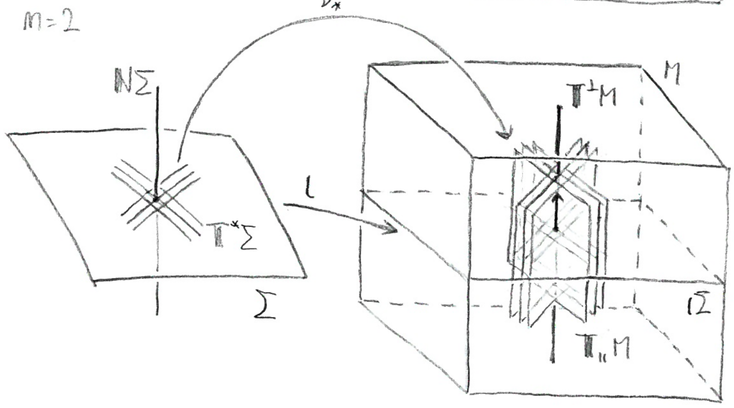


$\mathbb{T}_\parallel M$ tečné kovektory vnořené do $\mathbb{T}^*M$

$\nu_* : \mathbb{T}^*\Sigma \to \mathbb{T}_\parallel M \subset \mathbb{T}^*M$

definováno pomocí $^{\parallel}\delta$





2. fundamentální forma bez projektoru

∇ obecná kov. der. na $\mathbb{T}M$

$a, b \in \mathbb{T}\Sigma$ $\quad$ $\iota_* a, \iota_* b \in \mathbb{T}^\iota M$ $\quad$ tečné vektory

$\kappa, \lambda \in \mathbb{N}^*\Sigma$ $\quad$ $\nu_* \kappa, \nu_* \lambda \in \mathbb{T}_\perp M$ $\quad$ normálové formy

$\nu^*(\nabla_{\iota_* a} \iota_* b) \in \mathbb{N}\Sigma$

- ultralokální v a
  z vlastností ∇
- ultralokální v b

$$\nu^*(\nabla_{\iota_* a}(\iota_* f b)) = \nu^*(\iota_* f \nabla_{\iota_* a} \iota_* b + \iota_* b\, a[f]) = f\, \nu^*(\nabla_{\iota_* a} \iota_* b)$$

- lze reprezentovat tenzorem

$$\mathrm{II}_a \in \mathbb{T}^*\Sigma \otimes \mathbb{N}\Sigma \qquad \nu^*(\nabla_{\iota_* a} \iota_* b) = \mathrm{II}_a \cdot b$$

$$\mathrm{II} \in \mathbb{T}^0_2\Sigma \otimes \mathbb{N}\Sigma \qquad \mathrm{II}_a \cdot b = a \cdot \mathrm{II} \cdot b$$

$\iota^*(\nabla_{\iota_* a} \nu_* \kappa) \in \mathbb{T}^*\Sigma$

- ultralokální v a
  z vlastností ∇
- ultralokální v κ

$$\iota^*(\nabla_{\iota_* a} \nu_*(f\kappa)) = \iota^*(\iota_* f \nabla_{\iota_* a} \nu_* \kappa + \nu_* \kappa\, a[f]) = f\, \iota^*(\nabla_{\iota_* a} \nu_* \kappa)$$

- lze reprezentovat tenzorem II_a zavedeným výše

$$\mathrm{II}_a \in \mathbb{T}^*\Sigma \otimes \mathbb{N}\Sigma \qquad \iota^*(\nabla_{\iota_* a} \nu_* \kappa) = -\kappa \cdot \mathrm{II}_a$$

nechť $\iota^*(\nabla_{\iota_* a} \nu_* \kappa) = -\kappa \cdot \tilde{\mathrm{II}}_a$

$$0 = \nabla_{\iota_* a} \iota_*(\kappa \cdot b) = \nabla_{\iota_* a}(\nu_* \kappa \cdot \iota_* b) = (\nabla_{\iota_* a} \nu_* \kappa) \cdot (\iota_* b) + (\nu_* \kappa) \cdot \nabla_{\iota_* a} \iota_* b =$$
$$= \iota^*(\nabla_{\iota_* a} \nu_* \kappa) \cdot b + \kappa \cdot \nu^*(\nabla_{\iota_* a} \iota_* b) = -\kappa \cdot \tilde{\mathrm{II}}_a \cdot b + \kappa \cdot \mathrm{II}_a \cdot b \quad \Rightarrow \quad \tilde{\mathrm{II}}_a = \mathrm{II}_a$$

restrikce torze

$$T_{\iota_* a, \iota_* b} = \nabla_{\iota_* a} \iota_* b - \nabla_{\iota_* b} \iota_* a - [\iota_* a, \iota_* b] = \nabla_{\iota_* a} \iota_* b - \nabla_{\iota_* b} \iota_* a - \iota_*[a, b]$$

$$\nu^* T_{\iota_* a, \iota_* b} = \nu^*(\nabla_{\iota_* a} \iota_* b) - \nu^*(\nabla_{\iota_* b} \iota_* a) = a \cdot \mathrm{II} \cdot b - b \cdot \mathrm{II} \cdot a$$

$$\iota \nu^* T \equiv T^\nu_{\iota\iota} = \mathrm{II} - \mathrm{II}^T$$

Podvarieta s projektorem

$\iota : \Sigma \to M$ podvarieta Σ v M

přirozené struktury

$\iota_* : \mathbb{T}\Sigma \to \mathbb{T}^{\parallel}M \subset \mathbb{T}M$ vnoření tečných vektorů

$\iota^* : \mathbb{T}^*M \to \mathbb{T}^*\Sigma$ restrikce forem na tečné formy

$\nu_* : \mathbb{N}^*\Sigma \to \mathbb{T}_{\perp}M \subset \mathbb{T}^*M$ vnoření normálových forem

$\nu^* : \mathbb{T}M \to \mathbb{N}\Sigma$ restrikce vektorů na normál. vekt.

projektory na tečné a norm. složky

$\mathbb{T}^{\perp}M$ volba normál. podprostoru vektorů $\mathbb{T}^{\perp}M \subset \mathbb{T}M$ $\mathbb{T}M = \mathbb{T}^{\parallel}M \oplus \mathbb{T}^{\perp}M$

$\mathbb{T}_{\parallel}M$ volba tečného podprostoru forem $\mathbb{T}_{\parallel}M \subset \mathbb{T}^*M$ $\mathbb{T}^*M = \mathbb{T}_{\parallel}M \oplus \mathbb{T}_{\perp}M$

dualita $\mathbb{T}^{\perp}M$ a $\mathbb{T}_{\parallel}M$ – jeden prostor určuje druhý

$k \in \mathbb{T}^{\perp}M \iff \forall \alpha \in \mathbb{T}_{\parallel}M \quad \alpha \cdot k = 0$

$\alpha \in \mathbb{T}_{\parallel}M \iff \forall k \in \mathbb{T}^{\perp}M \quad \alpha \cdot k = 0$

projektory

$^{\parallel}\delta : \mathbb{T}M \to \mathbb{T}^{\parallel}M \quad \ker {^{\parallel}\delta} = \mathbb{T}^{\perp}M \quad \operatorname{img} {^{\parallel}\delta} = \mathbb{T}^{\parallel}M \qquad {^{\parallel}\delta} + {^{\perp}\delta} = \delta$

$^{\perp}\delta : \mathbb{T}M \to \mathbb{T}^{\perp}M \quad \ker {^{\perp}\delta} = \mathbb{T}^{\parallel}M \quad \operatorname{img} {^{\perp}\delta} = \mathbb{T}^{\perp}M \qquad {^{\parallel}\delta}\,{^{\perp}\delta} = {^{\perp}\delta}\,{^{\parallel}\delta} = 0$

$^{\parallel}\delta : \mathbb{T}^*M \to \mathbb{T}_{\parallel}M \quad \ker {^{\parallel}\delta} = \mathbb{T}_{\perp}M \quad \operatorname{img} {^{\parallel}\delta} = \mathbb{T}_{\parallel}M$

$^{\perp}\delta : \mathbb{T}M \to \mathbb{T}_{\perp}M \quad \ker {^{\perp}\delta} = \mathbb{T}_{\parallel}M \quad \operatorname{img} {^{\perp}\delta} = \mathbb{T}_{\perp}M$

výběr $\mathbb{T}^{\perp}M$, $\mathbb{T}_{\parallel}M$ umožňuje rozšířit isomorf. ι_* a ν_*

$\iota_* : \mathbb{T}\Sigma \to \mathbb{T}^{\parallel}M \subset \mathbb{T}M \qquad \iota_* a \in \mathbb{T}^{\parallel}M \qquad \nu^*\iota_* a = 0$

$\iota_* : \mathbb{N}\Sigma \to \mathbb{T}^{\perp}M \subset \mathbb{T}M \qquad \iota_* k \in \mathbb{T}^{\perp}M \qquad \nu^*\iota_* k = k$ rozšíření na $\mathbb{N}\Sigma$

$\iota_* : \mathbb{T}\oplus\mathbb{N}\Sigma \to \mathbb{T}M$ isomorfismus vektorů

$\nu_* : \mathbb{N}^*\Sigma \to \mathbb{T}_{\perp}M \subset \mathbb{T}M \qquad \nu_* k \in \mathbb{T}_{\perp}M \qquad \iota^*\nu_* k = 0$

$\nu_* : \mathbb{T}^*\Sigma \to \mathbb{T}_{\parallel}M \subset \mathbb{T}M \qquad \nu_*\alpha \in \mathbb{T}_{\parallel}M \qquad \iota^*\nu_*\alpha = \alpha$ rozšíření na $\mathbb{T}^*\Sigma$

$\nu_* : \mathbb{T}^*\oplus\mathbb{N}^*\Sigma \to \mathbb{T}^*M$ isomorfismus forem

$\iota\nu_* : (\mathbb{T}+\mathbb{N})^p_q\Sigma \to \mathbb{T}^p_q M$ isomorf. indukovaný ι_* na vekt. a ν_* na form.

při fixovaných podprostorech $\mathbb{T}^{\perp}M$ $\mathbb{T}_{\parallel}M$ resp. projektorech $^{\parallel}\delta$, $^{\perp}\delta$ netřeba rozlišovat mezi

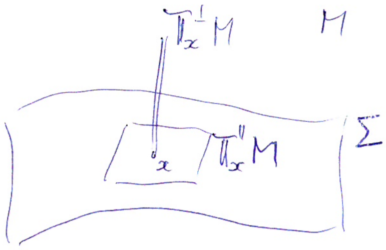
$$(\mathbb{T}+\mathbb{N})^p_q\,\Sigma \quad \text{a} \quad \mathbb{T}^p_q M$$

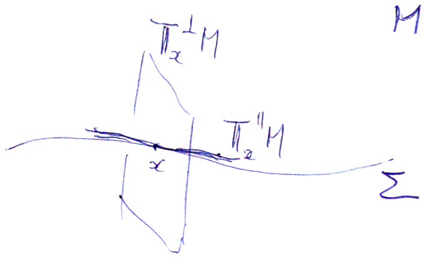
Podvariety riemanovské variety

$\iota: \Sigma \hookrightarrow M$

$\dim M = m$

$\dim N = n$





M s metrikou g

umožňuje ortogonální rozštěpení

$\mathbb{T}_x M = \mathbb{T}_x^\parallel M + \mathbb{T}_x^\perp M$

$\mathbb{T}_x^\parallel M = \mathbb{T}_x \Sigma$ tečné vektory

$\mathbb{T}_x^\perp M$ normálové vektory

$k \in \mathbb{T}_x^\perp M \iff \forall a \in \mathbb{T}_x^\parallel M \quad k \cdot g \cdot a = 0$

umožňuje vnořit $\mathbb{T}_x^* \Sigma$ do $\mathbb{T}^* M$ jako tečné formy

$\mathbb{T}_x^* M = \mathbb{T}_{x\parallel} M + \mathbb{T}_{x\perp} M$

$\mathbb{T}_{x\perp} M = \mathbb{N}_x \Sigma$ normálové 1-formy

$\mathbb{T}_{x\parallel} M$ tečné 1-formy

$\alpha \in \mathbb{T}_{x\parallel} M \iff \forall k \in \mathbb{T}_x^\perp M \quad \alpha \cdot k = 0$

$\iff \forall \kappa \in \mathbb{T}_{x\perp} M \quad \alpha \cdot g^{-1} \cdot \kappa = 0$

metrika se rozštěpuje na ortog. komponenty

$g = {}^\parallel g + {}^\perp g \qquad g^{-1} = {}^\parallel g^{-1} + {}^\perp g^{-1} \qquad \delta = {}^\parallel\delta + {}^\perp\delta \qquad {}^\parallel\delta, {}^\perp\delta$ projektory

konzistence zvedání/snižování indexů

$a = {}^\sharp\alpha \in \mathbb{T}^\parallel M \iff \alpha = {}^\flat a \in \mathbb{T}_\parallel M$

$k = {}^\sharp\kappa \in \mathbb{T}^\perp M \iff \kappa = {}^\flat k \in \mathbb{T}_\perp M$

g není smíšená komponenta $\quad a \cdot g \cdot k = 0 \quad a \in \mathbb{T}^\parallel M \quad k \in \mathbb{T}^\perp M$

Značení

$\delta = {}^{\parallel}\delta + {}^{\perp}\delta$

${}^{\parallel}\delta$ projektor na $T^{\parallel}M$

${}^{\perp}\delta$ projektor na $T^{\perp}M$

${}^{\parallel}A^{a\dots}_{b\dots} = {}^{\parallel}\delta^{a}_{k}\dots{}^{\parallel}\delta^{l}_{b}\dots A^{k\dots}_{l\dots}$ projekce na $T^{\parallel}M$ ve všech indexech

${}^{\perp}A^{a\dots}_{b\dots} = {}^{\perp}\delta^{a}_{k}\dots{}^{\perp}\delta^{l}_{b}\dots A^{k\dots}_{l\dots}$ projekce na $T^{\perp}M$ ve všech indexech

$A^{\dots\parallel a\dots}_{\dots\perp b\dots}$ či $A^{\dots\parallel\dots}_{\dots\perp\dots}$ smíšené projekce

$$A^{\dots\parallel a\dots}_{\dots\perp b\dots} = {}^{\parallel}\delta^{a}_{k}\,{}^{\perp}\delta^{l}_{b}\,A^{\dots k\dots}_{\dots l\dots}$$

${}^{\parallel}A$ resp. ${}^{\perp}A$ indikuje, že ${}^{\parallel}A$ je tečné, resp. ${}^{\perp}A$ je normální ve všech indexech

budeme užívat

$a, b, c, \dots \in T^{\parallel}M$

$\alpha, \beta, \gamma, \dots \in T_{\parallel}M$

$k, l, m, \dots \in T^{\perp}M$

$\kappa, \lambda, \mu, \dots \in T_{\perp}M$

metrika

${}^{\parallel}g = g_{\parallel\parallel}$ metrika na $T\Sigma \equiv T^{\parallel}M$

${}^{\perp}g = g_{\perp\perp}$ metrika v normálové bundle $N\Sigma \equiv T^{\perp}M$

${}^{\perp}g^{-1} = g^{-1}{}^{\perp\perp}$ metrika na $N^{*}\Sigma \equiv T_{\perp}M$

$g_{\perp\parallel} = g_{\parallel\perp} = 0$ ortogonalita

Rozštěpení kov. derivace na $\mathbb{T}M$

∇ obecná kov. der. na $\mathbb{T}M$

akce na tečném bundlu $\mathbb{T}\Sigma$

$$\nabla_c a = \overset{\shortparallel}{\nabla}_c a + \mathbb{I}_c \cdot a \qquad \text{Gaussova formule}$$

$$\overset{\shortparallel}{\nabla}_c a = {}^{\shortparallel}(\nabla_c a) \qquad \text{kov. der. na } \mathbb{T}\Sigma$$

$$\mathbb{I}_c \cdot a = c \cdot \mathbb{I} \cdot a = {}^{\perp}(\nabla_c a) \qquad \text{2. fund. forma}$$

$$\mathbb{I}_c = \mathbb{I}_{c\,\shortparallel}^{\perp} \qquad \mathbb{I}_c \in \mathbb{T}^{\perp}_{\shortparallel} M \leftrightarrow \mathbb{T}^* \otimes \mathbb{N}\Sigma$$

$$\mathbb{I} = \mathbb{I}^{\perp}_{\shortparallel\,\shortparallel} \qquad \mathbb{I} \in \mathbb{T}^{\perp}_{\shortparallel\shortparallel} M \leftrightarrow \mathbb{T}^0_2 \otimes \mathbb{N}\Sigma$$

důk: ${}^{\shortparallel}(\nabla_c a)$ splňuje všechny vlast. kov. der.

${}^{\perp}(\nabla_c a)$ ultralok. v a: ${}^{\perp}(\nabla_c (fa)) = {}^{\perp}(f\nabla_c a + c[f]\,a) = f\,{}^{\perp}(\nabla_c a)$

rozšíření $\overset{\shortparallel}{\nabla}$ na tečné tenzory $\mathbb{T}^{\shortparallel p}_{\shortparallel q} M \leftrightarrow \mathbb{T}^p_q \Sigma$

$$\overset{\shortparallel}{\nabla}_c A = {}^{\shortparallel}(\nabla_c A) \qquad \text{pro } A = {}^{\shortparallel}A$$

důk: stand. rozšíření na 1formy:

$(\overset{\shortparallel}{\nabla}_c \alpha)\cdot a = c[\alpha \cdot a] - \alpha \cdot \overset{\shortparallel}{\nabla}_c a = \nabla_c(\alpha \cdot a) - \alpha \cdot \nabla_c a = (\nabla_c \alpha)\cdot a = {}^{\shortparallel}(\nabla_c \alpha)\cdot a$

$\Rightarrow \overset{\shortparallel}{\nabla}_c \alpha = {}^{\shortparallel}(\nabla_c \alpha)$

komutace projekce s tenz. nás. + Leibniz $\Rightarrow$ rozšíření na tenzory

přirozeně platí

$$\overset{\shortparallel}{\nabla}_c {}^{\shortparallel}\delta = 0$$

důk: $\overset{\shortparallel}{\nabla}_c a = \overset{\shortparallel}{\nabla}_c({}^{\shortparallel}\delta \cdot a) = {}^{\shortparallel}(\nabla_c({}^{\shortparallel}\delta \cdot a)) = {}^{\shortparallel}((\nabla_c {}^{\shortparallel}\delta)\cdot a + {}^{\shortparallel}\delta \cdot \nabla_c a) = {}^{\shortparallel}(\nabla_c {}^{\shortparallel}\delta)\cdot a + {}^{\shortparallel}(\nabla_c a) = (\overset{\shortparallel}{\nabla}_c {}^{\shortparallel}\delta)\cdot a + \overset{\shortparallel}{\nabla}_c a$

rozštěpení torze

$$T^{\shortparallel}_{\shortparallel\shortparallel} = \mathbb{T}$$

$$T^{\perp}_{\shortparallel\shortparallel} = \mathbb{I} - \mathbb{I}^T$$

důk:

$$T^{\shortparallel}_{a,b} = {}^{\shortparallel}(\nabla_a b - \nabla_b a - [a,b]) = \overset{\shortparallel}{\nabla}_a b - \overset{\shortparallel}{\nabla}_b a - [a,b] = \mathbb{T}_{a,b}$$

$$T^{\perp}_{a,b} = {}^{\perp}(\nabla_a b - \nabla_b a - [a,b]) = a \cdot \mathbb{I} \cdot b - b \cdot \mathbb{I} \cdot a$$

akce na normálové bundle $\mathbb{N}\Sigma$

$$\nabla_c k = \overset{\perp}{\nabla}_c k - \tilde{\mathrm{II}}_c \cdot k \qquad \text{Weingartenova formule}$$

$\overset{\perp}{\nabla}_c k = {}^{\perp}(\nabla_c k)$ — kov. der. na $\mathbb{N}\Sigma$

$\tilde{\mathrm{II}}_c \cdot k = -{}^{\parallel}(\nabla_c k)$ — shape operátor $(S_k \cdot c \equiv \tilde{\mathrm{II}}_c \cdot k)$

$\tilde{\mathrm{II}}_c = \mathrm{II}_{c\perp}{}^{\parallel} \qquad \mathrm{II}_c \in \mathbb{T}^{\parallel}_{\perp} M \leftrightarrow \mathbb{T}\otimes\mathbb{N}^* \Sigma$

důkaz: ${}^{\perp}(\nabla_c k)$ splňuje všechny vl. kov. der. na $\mathbb{N}\Sigma$

${}^{\parallel}(\nabla_c k)$ ultralok. v k: ${}^{\parallel}(\nabla_c (f k)) = {}^{\parallel}(f \nabla_c k + c[f]k) = f\,{}^{\parallel}(\nabla_c k)$

rozšíření $\overset{\perp}{\nabla}$ na normálové tenzory $\mathbb{T}^{\perp p}_{\perp q} M \leftrightarrow \mathbb{N}^p_q \Sigma$

$$\overset{\perp}{\nabla}_c M = {}^{\perp}(\nabla_c M) \qquad \text{pro} \quad M = {}^{\perp}M$$

důkaz: stand. rozšíření na 1-formy:

$(\overset{\perp}{\nabla}_c \mu)\cdot k = c[\mu\cdot k] - \mu \cdot \overset{\perp}{\nabla}_c k = \nabla_c(\mu\cdot k) - \mu\cdot\nabla_c k = (\nabla_c \mu)\cdot k = {}^{\perp}(\nabla_c \mu)\cdot k$

$\Rightarrow \overset{\perp}{\nabla}_c \mu = {}^{\perp}(\nabla_c \mu)$

kombinace projekce a tenz. násobení $\Rightarrow$ rozšíření na norm. tenzory

přirozeně platí:

$$\overset{\perp}{\nabla}_c {}^{\perp}\delta = 0$$

restrikce ∇ na přizpůsobenou kov. der $\bar{\nabla}$

$\bar{\nabla} = \overset{\shortparallel}{\nabla} \oplus \overset{\perp}{\nabla}$ na $\mathbb{T}M \leftrightarrow (\mathbb{T} \oplus \mathbb{N})\Sigma$

$\bar{\nabla} = \overset{\shortparallel}{\nabla}$ na $\mathbb{T}^{\shortparallel}M \leftrightarrow \mathbb{T}\Sigma$

$\bar{\nabla} = \overset{\perp}{\nabla}$ na $\mathbb{T}^{\perp}M \leftrightarrow \mathbb{N}\Sigma$

přirozené rozšíření na $\mathbb{T}^p_q M \leftrightarrow (\mathbb{T} \oplus \mathbb{N})^p_q \Sigma$

$\bar{\nabla}\,{}^{\shortparallel}\delta = 0 \qquad \bar{\nabla}\,{}^{\perp}\delta = 0 \quad \Rightarrow \quad \bar{\nabla}$ přizpůsobená kov. der.

vztah ∇ a $\bar{\nabla}$

$\nabla = \bar{\nabla} + \mathbb{H}$ $\qquad$ $\mathbb{H}$ generované H

$H = \mathrm{II} - \bar{\mathrm{II}} \qquad H^{\perp}{}_{\shortparallel} = \mathrm{II} \qquad H^{\shortparallel}{}_{\perp} = -\bar{\mathrm{II}}$

$\Downarrow$

$\nabla_c a = \overset{\shortparallel}{\nabla}_c a + \mathrm{II}_c \cdot a \qquad \nabla_c k = \overset{\perp}{\nabla}_c k - \bar{\mathrm{II}}_c \cdot k$

$\nabla_c \alpha = \overset{\shortparallel}{\nabla}_c \alpha + \alpha \cdot \bar{\mathrm{II}}_c \qquad \nabla_c \kappa = \overset{\perp}{\nabla}_c \kappa - \kappa \cdot \mathrm{II}_c$

derivace projektorů

$\nabla_c {}^{\shortparallel}\delta = \mathrm{II}_c + \bar{\mathrm{II}}_c \quad \Leftarrow \quad \nabla_c {}^{\shortparallel}\delta = \bar{\nabla}_c {}^{\shortparallel}\delta + \mathrm{II}_c \cdot {}^{\shortparallel}\delta + {}^{\shortparallel}\delta \cdot \bar{\mathrm{II}}_c = \mathrm{II}_c + \bar{\mathrm{II}}_c$

$\nabla_c {}^{\perp}\delta = -(\mathrm{II}_c + \bar{\mathrm{II}}_c) \quad \Leftarrow \quad \nabla_c {}^{\perp}\delta = \bar{\nabla}_c {}^{\perp}\delta - \bar{\mathrm{II}}_c \cdot {}^{\perp}\delta - {}^{\perp}\delta \cdot \mathrm{II}_c = -(\mathrm{II}_c + \bar{\mathrm{II}}_c)$

křivost na $(\mathbb{T} + \mathbb{N})\Sigma$

$\bar{R} = \overset{\shortparallel}{R} + \overset{\perp}{R}$

$\overset{\shortparallel}{R} = \overset{\shortparallel}{R}_{\shortparallel\shortparallel\shortparallel}{}^{\shortparallel}{}_{\shortparallel} \qquad \overset{\perp}{R} = \overset{\perp}{R}_{\shortparallel\shortparallel\shortparallel}{}^{\perp}{}_{\perp} \qquad \bar{R} = \bar{R}_{\shortparallel\shortparallel\shortparallel}$

ortogonální rozštěpení a metrické kov. der.

$$g = {}^{\perp}g + {}^{\parallel}g \qquad \text{tj.} \qquad g_{\perp\parallel} = g_{\parallel\perp} = 0 \qquad T^{\parallel}M \perp T^{\perp}M$$

$$\nabla g = 0$$

$\Downarrow$

$\overset{\parallel}{\nabla}$ metrické na $T\Sigma$ $\qquad \overset{\parallel}{\nabla}{}^{\parallel}g = 0$

$\overset{\perp}{\nabla}$ metrické na $N\Sigma$ $\qquad \overset{\perp}{\nabla}{}^{\perp}g = 0$

$\bar{\nabla}$ metrické na $TM \leftrightarrow (T\oplus N)\Sigma$ $\qquad \bar{\nabla}g = 0$

$$g\cdot H_c + (g\cdot H_c)^T = 0 \qquad \text{tj.} \qquad H_{\underline{amn}} = -H_{\underline{anm}} \qquad H_{\underline{amn}} = g_{\underline{mk}}H_{\underline{a}}{}^{k}{}_{\underline{n}}$$

$${}^{\perp}g\cdot \mathrm{II} = \bar{\mathrm{II}}\cdot {}^{\parallel}g \qquad \text{tj.} \qquad \mathrm{II}_{\parallel\overset{\downarrow}{\perp}\parallel} = \bar{\mathrm{II}}_{\parallel\perp\overset{\downarrow}{\parallel}}$$

důkaz

$$\overset{\parallel}{\nabla}_c{}^{\parallel}g = {}^{\parallel}(\nabla_c{}^{\parallel}g) = {}^{\parallel}(\nabla_c({}^{\parallel}\delta\cdot g)) = {}^{\parallel}((\nabla_c{}^{\parallel}\delta)\cdot g) = {}^{\parallel}(\nabla_c{}^{\parallel}\delta)\cdot{}^{\parallel}g = (\overset{\parallel}{\nabla}_c{}^{\parallel}\delta)\cdot{}^{\parallel}g = 0$$

$$\overset{\perp}{\nabla}_c{}^{\perp}g = {}^{\perp}(\nabla_c{}^{\perp}g) = {}^{\perp}(\nabla_c({}^{\perp}\delta\cdot g)) = {}^{\perp}((\nabla_c{}^{\perp}\delta)\cdot g) = {}^{\perp}(\nabla_c{}^{\perp}\delta)\cdot{}^{\perp}g = (\overset{\perp}{\nabla}_c{}^{\perp}\delta)\cdot{}^{\perp}g = 0$$

$$\bar{\nabla}_c g = \overset{\parallel}{\nabla}_c{}^{\parallel}g + \overset{\perp}{\nabla}_c{}^{\perp}g = 0$$

$$0 = \nabla_c g = \bar{\nabla}_c g - g\,H_c - (g\,H_c)^T = -(g\,H_c + (g\,H_c)^T)$$

$$(g\,H_c + (g\,H_c)^T)_{\perp\parallel} = {}^{\perp}g\cdot H_c{}^{\perp}{}_{\parallel} + ({}^{\parallel}g\cdot H_c{}^{\parallel}{}_{\perp})^T = {}^{\perp}g\,\mathrm{II}_c - \bar{\mathrm{II}}_c\cdot{}^{\parallel}g = 0$$

derivace metrik

$$\nabla_c{}^{\parallel}g_{ab} = \mathrm{II}_{cab} + \mathrm{II}_{cba} \qquad \nabla_{\parallel c}{}^{\parallel}g_{ab} = \mathrm{II}_{cab} + \mathrm{II}_{cba}$$

$$\nabla_c{}^{\perp}g_{ab} = -\mathrm{II}_{cab} - \mathrm{II}_{cba} \qquad \nabla_{\parallel c}{}^{\perp}g_{ab} = -\mathrm{II}_{cab} - \mathrm{II}_{cba}$$

$$\nabla_c{}^{\parallel}g^{ab} = \mathrm{II}_c{}^{ab} + \mathrm{II}_c{}^{ba} \qquad \nabla_{\parallel c}{}^{\parallel}g^{ab} = \mathrm{II}_c{}^{ab} + \mathrm{II}_c{}^{ba}$$

$$\nabla_c{}^{\perp}g^{ab} = -\mathrm{II}_c{}^{ab} - \mathrm{II}_c{}^{ba} \qquad \nabla_{\parallel c}{}^{\perp}g^{ab} = -\mathrm{II}_c{}^{ab} - \mathrm{II}_c{}^{ba}$$

důkaz:

$$\nabla_{\parallel c}{}^{\parallel}g_{ab} = \bar{\nabla}_{\parallel c}{}^{\parallel}g_{ab} + H_c\,{}^{\parallel}g_{ab} = \mathrm{II}_{ca}{}^{k}\,{}^{\parallel}g_{kb} + \mathrm{II}_{cb}{}^{k}\,{}^{\parallel}g_{ak} = \mathrm{II}_{cab} + \mathrm{II}_{cba}$$

$$\nabla_{\parallel c}{}^{\perp}g_{ab} = \bar{\nabla}_{\parallel c}{}^{\perp}g_{ab} + H_c\,{}^{\perp}g_{ab} = -\mathrm{II}_c{}^{k}{}_{a}\,{}^{\perp}g_{kb} - \mathrm{II}_c{}^{k}{}_{b}\,{}^{\perp}g_{ak} = -\mathrm{II}_{cba} - \mathrm{II}_{cab}$$

obdobně ${}^{\parallel}g^{ab}$ a ${}^{\perp}g^{ab}$

Rozštěpení křivosti na $(T\!\!\parallel \oplus N\!\!\perp)\Sigma$

$$\nabla_{\parallel} = \bar{\nabla} + H\!\!\parallel \qquad \text{chápáno jako kov. der. na } \Sigma$$

$$H_c{}^{\perp}{}_{\parallel} = \mathrm{II}_c \qquad H_c{}^{\parallel}{}_{\perp} = -\bar{\mathrm{II}} \qquad H_c = \mathrm{II}_c - \bar{\mathrm{II}}_c \qquad \bar{\nabla}^{\perp}\delta = \bar{\nabla}^{\parallel}\delta = 0$$

$$R_{\parallel\parallel} = \bar{R} + \overset{\bar{\nabla}^{\parallel}}{d} H + H \wedge H$$

$$R_{\parallel a \parallel b}{}^{m}{}_{n} = \bar{R}_{ab}{}^{m}{}_{n} + (\overset{\bar{\nabla}^{\parallel}}{d}_a H_b)^{m}{}_{n} + (H_a \wedge H_b)^{m}{}_{n}$$

$$= \overset{\parallel}{R}_{ab}{}^{m}{}_{n} + \overset{\perp}{R}_{ab}{}^{m}{}_{n} + \bar{\nabla}_a H_b{}^{m}{}_{n} - \bar{\nabla}_b H_a{}^{m}{}_{n} + \overset{\#}{T}{}^{c}{}_{ab} H_c{}^{m}{}_{n} + H_a{}^{m}{}_{k} H_b{}^{k}{}_{n} - H_b{}^{m}{}_{k} H_a{}^{k}{}_{n}$$

$\Downarrow$

$$R_{\parallel a \parallel b}{}^{\parallel m}{}_{\parallel n} = \overset{\parallel}{R}_{ab}{}^{m}{}_{n} - \bar{\mathrm{II}}_{ak}{}^{m}\, \mathrm{II}_b{}^{k}{}_{n} + \bar{\mathrm{II}}_{bk}{}^{m}\, \mathrm{II}_a{}^{k}{}_{n} \qquad \text{Gaussova rovnice}$$

$$R_{\parallel a \parallel b}{}^{\perp m}{}_{\perp n} = \overset{\perp}{R}_{ab}{}^{m}{}_{n} - \mathrm{II}_a{}^{m}{}_{k}\, \bar{\mathrm{II}}_{bn}{}^{k} + \mathrm{II}_b{}^{m}{}_{k}\, \bar{\mathrm{II}}_{an}{}^{k} \qquad \text{Ricci(-Kühne) rov.}$$

$$R_{\parallel a \parallel b}{}^{\perp m}{}_{\parallel n} = \bar{\nabla}_a \mathrm{II}_b{}^{m}{}_{n} - \bar{\nabla}_b \mathrm{II}_a{}^{m}{}_{n} + \overset{\#}{T}{}^{c}{}_{ab}\, \mathrm{II}_c{}^{m}{}_{n} = (\overset{\bar{\nabla}^{\parallel}}{d}_a \mathrm{II}_b)^{m}{}_{n}$$

Codazzi-Mainardiho rov.

$$R_{\parallel a \parallel b}{}^{\parallel m}{}_{\perp n} = -\bar{\nabla}_a \bar{\mathrm{II}}_{bn}{}^{m} + \bar{\nabla}_b \bar{\mathrm{II}}_{an}{}^{m} - \overset{\#}{T}{}^{c}{}_{ab}\, \bar{\mathrm{II}}_{cn}{}^{m} = -(\overset{\bar{\nabla}^{\parallel}}{d}_a \bar{\mathrm{II}}_b)_{n}{}^{m}$$

metrická der.

$$\mathrm{II}_{a\overset{\downarrow}{m}n} = \bar{\mathrm{II}}_{am\overset{\downarrow}{n}}$$

$$R_{\parallel a \parallel b \parallel c \parallel d} = \overset{\parallel}{R}_{abcd} - (\mathrm{II}_a{}^{m}{}_{c}\, \mathrm{II}_b{}^{n}{}_{d} - \mathrm{II}_b{}^{m}{}_{c}\, \mathrm{II}_a{}^{n}{}_{d})\, {}^{\perp}g_{mn}$$

$$R_{\parallel a \parallel b}{}^{\perp m \perp n} = \overset{\perp}{R}_{ab}{}^{mn} - (\mathrm{II}_a{}^{m}{}_{c}\, \mathrm{II}_b{}^{n}{}_{d} - \mathrm{II}_b{}^{m}{}_{c}\, \mathrm{II}_a{}^{n}{}_{d})\, {}^{\parallel}g^{cd}$$

$$R_{\parallel a \parallel b}{}^{\perp n}{}_{\parallel n} = -R_{\parallel a \parallel b \parallel n}{}^{\perp m} = \bar{\nabla}_a \mathrm{II}_b{}^{m}{}_{n} - \bar{\nabla}_b \mathrm{II}_a{}^{m}{}_{n} + \overset{\#}{T}{}^{c}{}_{ab}\, \mathrm{II}_c{}^{m}{}_{n} = (\overset{\bar{\nabla}^{\parallel}}{d}_a \mathrm{II}_b)^{m}{}_{n}$$

Zúžení 2. fundamentální formy a křivosti

$$\mathrm{Tr}\,\mathrm{II}_m = \breve{\mathrm{II}}_{cm}{}^{c} \qquad \mathrm{Tr}\,\mathrm{II}_m = \mathrm{Tr}\,\mathrm{II}_{\perp m}$$

$$\overset{2}{\bar{\mathrm{II}}}_{ab} = \bar{\mathrm{II}}_{ak}{}^{c}\,\mathrm{II}^{k}{}_{bc} \qquad \overset{2}{\mathrm{II}}_{ab} = \overset{2}{\mathrm{II}}_{\|a\|b}$$

zúžení křivosti

$$R_{\|c\|a}{}^{\|c}{}_{\|b} = \overset{\|}{\mathrm{Ric}}_{ab} - \mathrm{Tr}\,\bar{\mathrm{II}}_k\,\mathrm{II}^{k}_{ab} + \overset{2}{\mathrm{II}}_{ab} - \bar{\mathrm{II}}_{ak}{}^{m}\,T^{\perp k}{}_{\|b\|m} = \mathrm{Ric}_{\|a\|b} - R_{\perp k\|a}{}^{\perp k}{}_{\|b}$$

$$R_{\|a\|b}{}^{\|c}{}_{\|c} = \mathrm{Tr}\,\overset{\|}{R}_{ab} - \overset{2}{\mathrm{II}}_{ab} + \overset{2}{\mathrm{II}}_{ba} = \mathrm{Tr}\,R_{\|a\|b} - R_{\|a\|b}{}^{\perp k}{}_{\perp k}$$

$$R_{\|a\|b}{}^{\perp k}{}_{\perp k} = \mathrm{Tr}\,\overset{\perp}{R}_{ab} + \overset{2}{\mathrm{II}}_{ab} - \overset{2}{\mathrm{II}}_{ba}$$

$$\mathrm{Tr}\,R_{\|a\|b} = \mathrm{Tr}\,\overset{\|}{R}_{ab} + \mathrm{Tr}\,\overset{\perp}{R}_{ab}$$

$$R_{\|c\|a}{}^{\|c}{}_{\perp n} = \overset{\perp}{\nabla}_a\,\mathrm{Tr}\,\mathrm{II}_n - \bar{\nabla}_c\,\bar{\mathrm{II}}_{an}{}^{c} + T^{c}{}_{ab}\,\bar{\mathrm{II}}_{cn}{}^{b} = \mathrm{Ric}_{\|a\perp n} - R_{\perp k\|a}{}^{\perp k}{}_{\perp n}$$

metrická derivace

$$\mathrm{II}_{ak\overset{\downarrow}{b}} = \breve{\mathrm{II}}_{ak\overset{\downarrow}{b}}$$

$$\mathrm{Tr}\,\mathrm{II}^{k} = \mathrm{II}^{k}_{ab}\,{}^{\|}g^{ab} \qquad n^2\varkappa^2 = (\mathrm{Tr}\,\mathrm{II})^2 = {}^{\perp}g^{kl}\,\mathrm{Tr}\,\mathrm{II}_k\,\mathrm{Tr}\,\mathrm{II}_l$$

$$\overset{2}{\mathrm{II}}_{ab} = \mathrm{II}^{k}_{a\,c}\,\mathrm{II}^{l}_{b\,d}\,{}^{\|}g^{cd}\,{}^{\perp}g_{kl} = \overset{2}{\mathrm{II}}_{ba} \qquad \mathrm{Tr}\,\overset{2}{\mathrm{II}} = {}^{\|}g^{ab}\,\overset{2}{\mathrm{II}}_{ab} = {}^{\perp}g_{kl}\,{}^{\|}g^{ab}\,{}^{\|}g^{cd}\,\mathrm{II}^{k}_{ac}\,\mathrm{II}^{l}_{bd}$$

zúžení křivosti – metrická bez torze

$$R_{\|c\|a}{}^{\|c}{}_{\|b} = \overset{\|}{\mathrm{Ric}}_{ab} - \mathrm{Tr}\,\mathrm{II}_k\,\mathrm{II}^{k}_{ab} + \overset{2}{\mathrm{II}}_{ab} = \mathrm{Ric}_{\|a\|b} - R_{\perp k\|a}{}^{\perp k}{}_{\|b}$$

$$R_{\|c\|a}{}^{\|c}{}_{\perp n} = \overset{\perp}{\nabla}_a\,\mathrm{Tr}\,\mathrm{II}_n - \breve{\nabla}_c\,\bar{\mathrm{II}}_{an}{}^{c} = \mathrm{Ric}_{\|a\perp n} - R_{\perp k\|a}{}^{\perp k}{}_{\perp n} = \mathrm{Ric}_{\|a\perp n} - R_{\perp k\perp n}{}^{\perp k}{}_{\|a}$$

$$R_{\|a\|b}{}^{\|a\|b} = \overset{\|}{R} - (\mathrm{Tr}\,\mathrm{II})^2 + \mathrm{Tr}\,\overset{2}{\mathrm{II}} = R - 2\,\mathrm{Ric}_{\perp k}{}^{\perp k} + R_{\perp k\perp l}{}^{\perp k\perp l}$$

Semi-umbilic splitting of general cov. der.

$$\overline{\mathrm{II}}_{ak}{}^{b} = \frac{1}{M}\,\mathrm{Tr}\,\mathrm{II}_k\,{}^{\parallel}\delta_a^b$$

$$\mathrm{II}^2_{ab} = \overline{\mathrm{II}}_{ak}{}^{c}\,\mathrm{II}^k{}_{bc} = \frac{1}{M}\,\mathrm{Tr}\,\mathrm{II}_k\,\mathrm{II}^k{}_{ba}$$

curvature

$$R_{\parallel a\parallel b}{}^{\parallel m}{}_{\parallel n} = \overset{\parallel}{R}_{ab}{}^{m}{}_{n} - \frac{1}{M}\,\mathrm{Tr}\,\mathrm{II}_k\left({}^{\parallel}\delta^m_a\,\mathrm{II}^k{}_{bn} - {}^{\parallel}\delta^m_b\,\mathrm{II}^k{}_{an}\right) = \overset{\parallel}{R}_{ab}{}^{m}{}_{n} + \mathrm{II}^2_{na}\,{}^{\parallel}\delta^m_b - \mathrm{II}^2_{nb}\,{}^{\parallel}\delta^m_a$$

$$R_{\parallel a\parallel b}{}^{\perp m}{}_{\perp n} = \overset{\perp}{R}_{ab}{}^{m}{}_{n} - \frac{1}{M}\,\mathrm{Tr}\,\mathrm{II}_n\,T^{\perp m}{}_{\parallel a\parallel b}$$

$$R_{\parallel a\parallel b}{}^{\perp m}{}_{\parallel n} = \bar{\nabla}_a\mathrm{II}_b{}^{m}{}_{n} - \bar{\nabla}_b\mathrm{II}_a{}^{m}{}_{n} + \mathbb{T}^c_{ab}\,\mathrm{II}_c{}^{m}{}_{n} = \left(\bar{\bar{d}}_a\mathrm{II}_b\right)^m{}_n$$

$$R_{\parallel a\parallel b}{}^{\parallel m}{}_{\perp n} = \frac{1}{M}\left({}^{\parallel}\delta^m_a\,\overset{\perp}{\nabla}_b\,\mathrm{Tr}\,\mathrm{II}_n - {}^{\parallel}\delta^m_b\,\overset{\perp}{\nabla}_a\,\mathrm{Tr}\,\mathrm{II}_n - \mathbb{T}^m_{ab}\,\mathrm{Tr}\,\mathrm{II}_n\right)$$

contraction of curvature

$$R_{\parallel c\parallel a}{}^{\parallel c}{}_{\parallel b} = \overset{\parallel}{\mathrm{Ric}}_{ab} - \frac{M-1}{M}\,\mathrm{Tr}\,\mathrm{II}_k\,\mathrm{II}^k{}_{ab} = \overset{\parallel}{\mathrm{Ric}}_{ab} - (M-1)\,\mathrm{II}^2_{ba}$$

$$R_{\parallel a\parallel b}{}^{\parallel m}{}_{\parallel m} = \mathrm{Tr}\,\overset{\parallel}{R}_{ab} + \frac{1}{M}\,\mathrm{Tr}\,\mathrm{II}_k\,T^{\perp k}{}_{\parallel a\parallel b} = \mathrm{Tr}\,\overset{\parallel}{R}_{ab} - \mathrm{II}^2_{ab} + \mathrm{II}^2_{ba}$$

$$R_{\parallel a\parallel b}{}^{\perp m}{}_{\perp m} = \mathrm{Tr}\,\overset{\perp}{R}_{ab} - \frac{1}{M}\,\mathrm{Tr}\,\mathrm{II}_k\,T^{\perp k}{}_{\parallel a\parallel b} = \mathrm{Tr}\,\overset{\perp}{R}_{ab} + \mathrm{II}^2_{ab} - \mathrm{II}^2_{ba}$$

$$\mathrm{Tr}\,R_{\parallel a\parallel b} = \mathrm{Tr}\,\overset{\parallel}{R}_{ab} + \mathrm{Tr}\,\overset{\perp}{R}_{ab}$$

$$R_{\parallel c\parallel a}{}^{\parallel c}{}_{\perp n} = \frac{M-1}{M}\,\overset{\perp}{\nabla}_a\,\mathrm{Tr}\,\mathrm{II}_n - \frac{1}{M}\,\mathbb{T}^c_{ca}\,\mathrm{Tr}\,\mathrm{II}_n$$

Semi-umbilic splitting of torsion-free cov. der

$$\widetilde{\mathrm{II}}_{ak}{}^{b} = \frac{1}{m}\,\mathrm{Tr}\,\mathrm{II}_k\,{}^{\parallel}\delta_a^b \qquad T^c_{ab} = 0$$

$$\overset{2}{\mathrm{II}}_{ab} = \widetilde{\mathrm{II}}_{ak}{}^{c}\,\mathrm{II}_b{}^{k}{}_{c} = \frac{1}{m}\,\mathrm{Tr}\,\mathrm{II}_k\,\mathrm{II}^k_{ab} \qquad \overset{2}{\mathrm{II}}_{ab} = \overset{2}{\mathrm{II}}_{ba} \quad \Leftarrow \quad \mathrm{II}^k_{ab} = \mathrm{II}^k_{ba}$$

curvature

$$R_{\parallel a\parallel b}{}^{\parallel m}{}_{\parallel n} = \overset{\parallel}{R}_{ab}{}^{m}{}_{n} - \frac{1}{m}\,\mathrm{Tr}\,\mathrm{II}_k\left({}^{\parallel}\delta^m_a\,\mathrm{II}^k_{bn} - {}^{\parallel}\delta^m_b\,\mathrm{II}^k_{an}\right) = \overset{\parallel}{R}_{ab}{}^{m}{}_{n} - {}^{\parallel}\delta^m_a\,\overset{2}{\mathrm{II}}_{bn} + {}^{\parallel}\delta^m_b\,\overset{2}{\mathrm{II}}_{an}$$

$$R_{\parallel a\parallel b}{}^{\perp m}{}_{\perp n} = \overset{\perp}{R}_{ab}{}^{m}{}_{n}$$

$$R_{\parallel a\parallel b}{}^{\perp m}{}_{\parallel n} = \widetilde{\nabla}_a\,\mathrm{II}_b{}^{m}{}_{n} - \widetilde{\nabla}_b\,\mathrm{II}_a{}^{m}{}_{n}$$

$$R_{\parallel a\parallel b}{}^{\parallel m}{}_{\perp n} = \frac{1}{m}\left({}^{\parallel}\delta^m_a\,\overset{\perp}{\nabla}_b\,\mathrm{Tr}\,\mathrm{II}_n - {}^{\parallel}\delta^m_b\,\overset{\perp}{\nabla}_a\,\mathrm{Tr}\,\mathrm{II}_n\right)$$

contraction of curvature

$$R_{\parallel c\parallel a}{}^{\parallel c}{}_{\parallel b} = \overset{\parallel}{Ric}_{ab} - \frac{n-1}{m}\,\mathrm{Tr}\,\mathrm{II}_k\,\mathrm{II}^k_{ab} = \overset{\parallel}{Ric}_{ab} - (n-1)\,\overset{2}{\mathrm{II}}_{ab}$$

$$R_{\parallel a\parallel b}{}^{\parallel m}{}_{\parallel m} = \mathrm{Tr}\,\overset{\parallel}{R}_{ab} \qquad R_{\parallel a\parallel b}{}^{\perp m}{}_{\perp m} = \mathrm{Tr}\,\overset{\perp}{R}_{ab} \qquad \mathrm{Tr}\,R_{\parallel a\parallel b} = \mathrm{Tr}\,\overset{\parallel}{R}_{ab} + \mathrm{Tr}\,\overset{\perp}{R}_{ab}$$

$$R_{\parallel c\parallel a}{}^{\parallel c}{}_{\perp n} = \frac{n-1}{m}\,\overset{\perp}{\nabla}_a\,\mathrm{Tr}\,\mathrm{II}_n$$

Totally umbilic submanifolds

metric g_{ab}, Levi-Civita der ∇_a $\qquad \nabla g = 0 \qquad T = 0$

$\mathrm{II}_{akb}^{\perp} = \overline{\mathrm{II}}_{akb}^{\perp} \qquad \mathrm{II}_{ab}^{\ \ k} = \mathrm{II}_{ba}^{\ \ k} \qquad \mathrm{II}_{ab}^{?} = \mathrm{II}_{ba}^{?}$

umbilic $\qquad \overline{\mathrm{II}}_{ak}^{\ \ b} = \frac{1}{m} \mathrm{Tr}\,\mathrm{II}_k \, {}^{\parallel}\delta_c^b \qquad \mathrm{II}_{ab}^{\ \ k} = \frac{1}{m} \mathrm{Tr}\,\mathrm{II}^k \, {}^{\parallel}g_{ab}$

$m^2 \varkappa^2 = (\mathrm{Tr}\,\mathrm{II})^2 = {}^{\perp}g^{kl}\, \mathrm{Tr}\,\mathrm{II}_k \, \mathrm{Tr}\,\mathrm{II}_l \qquad \mathrm{II}_{ab}^{2} = \frac{1}{m^2}(\mathrm{Tr}\,\mathrm{II})^2 \, {}^{\parallel}g_{ab} = \varkappa^2 \, {}^{\parallel}g_{ab}$

curvature

$$R_{\parallel a \parallel b \parallel c \parallel d} = \overset{\parallel}{R}_{abcd} - \varkappa^2 \left({}^{\parallel}g_{ac} \, {}^{\parallel}g_{bd} - {}^{\parallel}g_{ad} \, {}^{\parallel}g_{bc} \right)$$

$$R_{\parallel a \parallel b}{}^{\perp m}{}_{\perp n} = \overset{\perp}{R}_{ab}{}^{m}{}_{n}$$

$$R_{\parallel a \parallel b}{}^{\perp m}{}_{\parallel c} = \frac{1}{m} \left(\overset{\perp}{\nabla}_a \mathrm{Tr}\,\mathrm{II}^m \, {}^{\parallel}g_{bc} - \overset{\perp}{\nabla}_b \mathrm{Tr}\,\mathrm{II}^m \, {}^{\parallel}g_{ac} \right)$$

$$R_{\parallel a \parallel b \parallel c \perp n} = \frac{1}{m} \left({}^{\parallel}g_{ca} \, \overset{\perp}{\nabla}_b \mathrm{Tr}\,\mathrm{II}_n - {}^{\parallel}g_{cb} \, \overset{\perp}{\nabla}_a \mathrm{Tr}\,\mathrm{II}_n \right)$$

contractions of curvature

$$R_{\parallel c \parallel a}{}^{\parallel c}{}_{\parallel b} = \mathrm{Ric}_{ab} - (m-1)\varkappa^2 \, {}^{\parallel}g_{ab} = \mathrm{Ric}_{\parallel a \parallel b} - R_{\perp k \parallel a}{}^{\perp k}{}_{\parallel b}$$

$$R_{\parallel c \parallel a}{}^{\parallel c}{}_{\perp n} = \frac{m-1}{m} \overset{\perp}{\nabla}_a \mathrm{Tr}\,\mathrm{II}_n = \mathrm{Ric}_{\parallel a \perp n} - R_{\perp k \parallel a}{}^{\perp k}{}_{\perp n} = \mathrm{Ric}_{\parallel a \perp n} - R_{\perp k \perp n}{}^{\perp k}{}_{\parallel a}$$

$$R_{\parallel a \parallel b}{}^{\parallel m}{}_{\parallel m} = \mathrm{Tr}\,\overset{\parallel}{R}_{ab} = 0 \qquad R_{\parallel a \parallel b}{}^{\perp m}{}_{\perp m} = \mathrm{Tr}\,\overset{\perp}{R}_{ab} = 0 \qquad \mathrm{Tr}\,R_{ab} = 0$$

$$R_{\parallel a \parallel b}{}^{\parallel a \parallel b} = \overset{\parallel}{R} - m(m-1)\varkappa^2 = R - 2\,\mathrm{Ric}_{\perp k}{}^{\perp k} + R_{\perp k \perp l}{}^{\perp k \perp l}$$

Submanifold in Einstein space

$$Ric_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab} = 0 \qquad \Lambda = \frac{(m-1)(m-2)}{2L^2}$$

$$\Downarrow$$

$$Ric_{ab} = \frac{(m-1)}{L^2} g_{ab} \qquad R = \frac{m(m-1)}{L^2}$$

$$^{\shortparallel}Ric_{ab} = Ric_{\shortparallel a \shortparallel b} - R_{\perp k \shortparallel a}{}^{\perp k}{}_{\shortparallel b} + Tr \mathrm{II}_k \mathrm{II}^k_{ab} - \mathrm{II}^2_{ab}$$

$$= \frac{m-1}{L^2} {}^{\shortparallel}g_{ab} - R_{\perp k \shortparallel a}{}^{\perp k}{}_{\shortparallel b} + Tr \mathrm{II}_k \mathrm{II}^k_{ab} - \mathrm{II}^2_{ab}$$

$$\frac{1}{n(n-1)} {}^{\shortparallel}R = \frac{1}{L^2} + \frac{1}{n(n-1)} \left((Tr\mathrm{II})^2 - Tr\mathrm{II}^2 \right) + \frac{1}{n(n-1)} \underbrace{\left(R_{\perp k \perp l}{}^{\perp k \perp l} - \frac{(m-n)(m-n-1)}{L^2} \right)}$$

1) $m = n+1$

2) $R_{\perp\perp\perp\perp} = \frac{1}{2L^2} {}^{\perp}g \wedge {}^{\perp}g$ $\Rightarrow 0$

umbilic

$$Tr\mathrm{II}_k \mathrm{II}^k_{ab} - \mathrm{II}^2_{ab} = (n-1) \varkappa^2 \, {}^{\shortparallel}g_{ab}$$

$$\frac{1}{n(n-1)} \left((Tr\mathrm{II})^2 - Tr\mathrm{II}^2 \right) = \varkappa^2$$

Submanifold in maximally symmetric space

$g \quad \nabla \qquad \nabla g = 0 \quad T = 0$

$\mathrm{II}_{akb} = \overline{\mathrm{II}}_{akb} \qquad \mathrm{II}^{k}_{ab} = \mathrm{II}^{k}_{ba} \qquad \mathrm{II}^{2}_{ab} = \mathrm{II}^{2}_{ba} \qquad \mathcal{H}^2 = \frac{1}{M^2}(\mathrm{Tr}\,\mathrm{II})^2$

maximally symmetric space

$$R_{abcd} = \frac{1}{L^2}\left(g_{ac}g_{bd} - g_{ad}g_{bc}\right) \qquad R = \frac{1}{L^2}\frac{1}{2}\, g \wedge g$$

$$Ric_{ab} = \frac{m-1}{L^2}\, g_{ab}$$

$$R = \frac{m(m-1)}{L^2} = \text{const}$$

curvature splitting

$${}^{\parallel}R_{abcd} = \frac{1}{L^2}\left({}^{\parallel}g_{ac}\,{}^{\parallel}g_{bd} - {}^{\parallel}g_{ad}\,{}^{\parallel}g_{bc}\right) + \left(\mathrm{II}^{k}_{ac}\,\mathrm{II}^{l}_{bd} - \mathrm{II}^{k}_{ad}\,\mathrm{II}^{l}_{bc}\right){}^{\perp}g_{kl}$$

$${}^{\perp}R_{ab}{}^{mn} = \left(\mathrm{II}^{m}_{ac}\,\mathrm{II}^{n}_{bd} - \mathrm{II}^{m}_{bc}\,\mathrm{II}^{n}_{ad}\right){}^{\parallel}g^{cd}$$

$$\overline{\nabla}_a \mathrm{II}^{m}_{bn} = \overline{\nabla}_b \mathrm{II}^{m}_{an}$$

$${}^{\parallel}Ric_{ab} = \frac{m-1}{L^2}\,{}^{\parallel}g_{ab} + \mathrm{Tr}\,\mathrm{II}_k\,\mathrm{II}^{k}_{ab} - \mathrm{II}^{2}_{ab}$$

$${}^{\perp}\nabla_a \mathrm{Tr}\,\mathrm{II}^{m} = \overline{\nabla}_n \mathrm{II}_a{}^{mn}$$

$$\frac{1}{m(m-1)}\,{}^{\parallel}R = \frac{1}{L^2} + \frac{1}{m(m-1)}\left((\mathrm{Tr}\,\mathrm{II})^2 - \mathrm{Tr}\,\mathrm{II}^2\right)$$

Totally umbilic submanifold of maximally sym. space

$g \quad \nabla \qquad \nabla g = 0 \quad T = 0$

$\mathbb{I}_{akb} = \overline{\mathbb{I}}_{akb} \qquad \mathbb{I}^{k}_{ab} = \mathbb{I}^{k}_{ba} \qquad \mathbb{I}^{?}_{ab} = \mathbb{I}^{?}_{ba}$

umbilic

$\mathbb{I}^{k}_{ab} = \frac{1}{m} \operatorname{Tr}\mathbb{I}^{k}\, {}^{\shortparallel}g_{ab} \qquad (\operatorname{Tr}\mathbb{I})^2 = m^2 \varkappa^2 \qquad \mathbb{I}^{?}_{ab} = \varkappa^{?}\, {}^{\shortparallel}g_{ab} \quad \operatorname{Tr}\mathbb{I}^{2} = m\varkappa^2$

maximally symetric space

$R_{abcd} = \frac{1}{L^2}(g_{ac}g_{bd} - g_{ad}g_{bc})$

$Ric_{ab} = \frac{m-1}{L^2} g_{ab}$

$R = \frac{m(m-1)}{L^2} = \text{const}$

curvature splitting

$\overset{\shortparallel}{R}_{abcd} = \left(\frac{1}{L^2} + \varkappa^2\right)\left({}^{\shortparallel}g_{ac}\,{}^{\shortparallel}g_{bd} - {}^{\shortparallel}g_{ad}\,{}^{\shortparallel}g_{bc}\right)$

maximally symmetric $\qquad \frac{1}{\ell^2} = \frac{1}{L^2} + \varkappa^2 = \text{const}$

$\overset{\perp}{R}_{ab}{}^{m}{}_{n} = 0$

$\overset{\perp}{\nabla}_a \operatorname{Tr}\mathbb{I}_n = 0$

$\overset{\shortparallel}{Ric}_{ab} = \frac{m-1}{\ell^2}\, {}^{\shortparallel}g_{ab}$

$\overset{\shortparallel}{R} = \frac{m(m-1)}{\ell^2}$

$\frac{1}{\ell^2} = \frac{1}{L^2} + \varkappa^2$

Vnoření nadplochy

$\dim \mathbb{N} \Sigma = 1 \qquad \dim M = \dim \Sigma + 1$

normalizované normále

ν normaliz. normálová forma
n normaliz. normálový vektor

} duální báze $n \cdot \nu = 1$

$^{\perp}\delta = n\,\nu \qquad {}^{\parallel}\delta = \delta - {}^{\perp}\delta$

metrika na norm. bundle

$^{\perp}g = s\,\nu\,\nu \qquad \nu = s\,{}^{\perp}g \cdot n \qquad n = s\,{}^{\perp}g^{-1} \cdot \nu \qquad s = \pm 1$

zkrácené značení

$A^{\dots\perp\dots} = A^{\dots k \dots}\,\nu_k \qquad A_{\dots\perp\dots} = A_{\dots k \dots}\,n^k \qquad A_{\cdot\perp\cdot} = s\,A^{\cdot\perp\cdot}$

obecně nemáme metriku $^{\parallel}g$ na $T\Sigma$

∇ obecná normálově plochá kov. der. na TM

$\overset{\perp}{\nabla} n = 0 \qquad \overset{\perp}{\nabla} \nu = 0 \qquad$ tj. $^{\perp}(\nabla_{\parallel} n) = 0 \qquad {}^{\perp}(\nabla_{\parallel} \nu) = 0$

$\Downarrow$

$\overset{\perp}{\nabla}\,{}^{\perp}g = 0 \qquad \overset{\perp}{R} = 0$

alternativní zavedení:

předpokládáme pouze projektory $^{\perp}\delta$ $^{\parallel}\delta$, ne normalizovanou norm. ν

∇ normálově plochá kov. der., tj. $\overset{\perp}{R} = 0$

$\Rightarrow$ existuje kov. konst. norm. vekt. n $\quad \overset{\perp}{\nabla} n = 0$

volba škály (v jednom bodě) definuje normály n, ν

vnější křivost

$$\mathrm{II}_{ab}{}^{k} = -K_{ab}\, n^{k} \qquad K_{ab} = -\mathrm{II}_{ab}{}^{k}\, \nu_k \qquad \mathrm{II}^2_{ab} = K_a{}^m K_{bm}$$

$$\bar{\mathrm{II}}_{ak}{}^{b} = -K_a{}^b\, \nu_k \qquad K_a{}^b = -\bar{\mathrm{II}}_{ak}{}^{b}\, n^k \qquad -\mathrm{Tr}\,\mathrm{II}_m = \mathcal{K}\, \nu_m \quad \mathcal{K} = K_a{}^a = -\mathrm{Tr}\,\mathrm{II}_1$$

$$\mathcal{H}^2 = s\left(\frac{\mathcal{K}}{m}\right)^2$$

derivace normály podél Σ

$$\nabla_{\parallel a}\nu_b = K_{ab} \qquad \Leftarrow \nabla_{\parallel a}\nu_b = \bar{\nabla}_a \nu_b - H_{ab}^{k}\nu_k = -\mathrm{II}_{ab}{}^{k}\nu_k = K_{ab}$$

$$\nabla_{\parallel a} n^b = K_a{}^b \qquad \Leftarrow \nabla_{\parallel a} n^b = \bar{\nabla}_a n^b + H_{ak}^{b} n^k = -\bar{\mathrm{II}}_{ak}{}^{b} n^k = K_a{}^b$$

derivace projektorů podél Σ

$$\nabla_{\parallel a}{}^{\perp}\delta_b^c = -\nabla_{\parallel a}{}^{\parallel}\delta_b^c = K_{ab}\, n^c + K_a{}^c \nu_b \qquad \Leftarrow \nabla_{\parallel a}{}^{\perp}\delta_b^c = \nabla_{\parallel a}(n^c\nu_b) = K_{ab}n^c + K_a{}^c\nu_b$$

$$(\nabla_{\parallel a}{}^{\perp}\delta)^{\perp}_{\parallel b} = K_{ab} \qquad (\nabla_{\parallel a}{}^{\perp}\delta)^{\parallel b}_{\perp} = K_a{}^b$$

projekce torze

$$T_{\parallel a \parallel b}{}^{\parallel c} = \overset{\parallel}{T}{}^c_{ab}$$

$$T_{\parallel a \parallel b}{}^{\perp} = -K_{ab} + K_{ba} \qquad \Leftarrow T_{\parallel a \parallel b}{}^{\perp} = \mathrm{II}^{\perp}_{ab} - \mathrm{II}^{\perp}_{ba}$$

rozštěpení křivosti

$$R_{\parallel a \parallel b}{}^{\parallel m}{}_{\parallel n} = \overset{\parallel}{R}{}_{ab}{}^{m}{}_{n} - K_a{}^m K_{bn} + K_b{}^m K_{an}$$

$$R_{\parallel a \parallel b}{}^{\perp}{}_{\perp} = -K_{ak}K_b{}^k + K_{bk}K_a{}^k$$

$$R_{\parallel a \parallel b}{}^{\perp}{}_{\parallel c} = -(\overset{\parallel}{\nabla}\mathrm{d}_a K_b)_c = -\overset{\parallel}{\nabla}_a K_{bc} + \overset{\parallel}{\nabla}_b K_{ac} - \overset{\parallel}{T}{}^n_{ab} K_{nc}$$

$$R_{\parallel a \parallel b}{}^{\parallel c}{}_{\perp} = (\overset{\parallel}{\nabla}\mathrm{d}_a K_b)^c = \overset{\parallel}{\nabla}_a K_b{}^c - \overset{\parallel}{\nabla}_b K_a{}^c + \overset{\parallel}{T}{}^n_{ab} K_n{}^c$$

zúžená křivost

$$R_{\parallel c \parallel a}{}^{\parallel c}{}_{\parallel b} = \overset{\parallel}{Ric}_{ab} - \mathcal{K} K_{ab} + K_a{}^c K_{cb}$$

$$R_{\parallel a \parallel b}{}^{\parallel c}{}_{\parallel c} = \mathrm{Tr}\overset{\parallel}{R}_{ab} - K_a{}^m K_{bm} + K_b{}^m K_{am}$$

$$R_{\parallel a \parallel b}{}^{\perp m}{}_{\perp m} = K_a{}^m K_{bm} - K_b{}^m K_{am}$$

$$\mathrm{Tr}R_{\parallel a \parallel b} = \mathrm{Tr}\overset{\parallel}{R}_{ab}$$

$$R_{\parallel c \parallel a}{}^{\parallel c}{}_{\perp} = \overset{\parallel}{\nabla}_c K_a{}^c - \overset{\parallel}{\nabla}_a \mathcal{K} + K_m{}^n \overset{\parallel}{T}{}^m_{na}$$

Metrické vnoření nadplochy

metrika na M

$$g = s\,\nu\nu + q \qquad {}^{\perp}g = s\,\nu\nu \qquad {}^{\parallel}g = q \qquad s = \pm 1 \qquad s^2 = 1$$

metrická derivace

$$\nabla g = 0 \;\Rightarrow\; \overset{\parallel}{\nabla} q = 0 \qquad \overset{\perp}{\nabla}\nu = 0 \qquad \overset{\perp}{R} = 0 \qquad \text{obecná } T$$

$$\mathrm{II}_{akb} = \overline{\mathrm{II}}_{akb} \qquad K_{ab} = s\,K_{ab}$$

rozštěpení křivosti

$$R_{\parallel a\parallel b\parallel c\parallel d} = \overset{\parallel}{R}_{abcd} - s\,(K_{ac}K_{bd} - K_{ad}K_{bc})$$

$$R_{\parallel a\parallel b\parallel c\perp} = (\overset{\parallel}{\mathrm{d}}_a K_b)_c = \overset{\parallel}{\nabla}_a K_{bc} - \overset{\parallel}{\nabla}_b K_{ac} + \overset{\parallel}{T}{}^m_{ab} K_{mc}$$

Levi-Civitova derivace

$$T^c_{ab} = 0 \qquad K_{ab} = K_{ba} = s\,K_{ab} \qquad K^2_{ab} = K_{ac}K_{bd}\,q^{cd} = s\,\mathrm{II}^2_{ab}$$

$$k = K_a{}^a = -\mathrm{Tr}\,\mathrm{II}_\perp \qquad m^2\varkappa^2 = s\,k^2 \qquad K^2 = K^2{}_a{}^a = s\,\mathrm{Tr}\,\mathrm{II}^2$$

rozštěpení křivosti

$$R_{\parallel a\parallel b\parallel c\parallel d} = \overset{\parallel}{R}_{abcd} - s\,(K_{ac}K_{bd} - K_{ad}K_{bc})$$

$$R_{\parallel a\parallel b\parallel c\perp} = \overset{\parallel}{\nabla}_c K_{bc} - \overset{\parallel}{\nabla}_b K_{ac}$$

$$R_{\parallel c\parallel a}{}^{\parallel c}{}_{\parallel b} \equiv Ric_{\parallel a\parallel b} - s\,R_{\perp\parallel c\perp\parallel b} = \overset{\parallel}{Ric}_{ab} - s\,(k K_{ab} - K^2_{ab})$$

$$R_{\parallel c\parallel a}{}^{\parallel c}{}_{\perp} = Ric_{\perp\parallel a} = \overset{\parallel}{\nabla}_c K_a{}^c - \overset{\parallel}{\nabla}_a k$$

$$R_{\parallel a\parallel b}{}^{\parallel a\parallel b} = R - 2s\,Ric_{\perp\perp} = \overset{\parallel}{R} - s\,(k^2 - K^2)$$

Gauss-Codazziho identita

$$R = \overset{\parallel}{R} + 2s\,Ric_{\perp\perp} - s\,(k^2 - K^2)$$

normálové složky Einsteinova tenzoru

$$Ric_{\perp\perp} = \frac{s}{2}(R - \overset{\parallel}{R}) + \frac{1}{2}(k^2 - K^2)$$

$$Ein_{\perp\perp} = Ric_{\perp\perp} - \frac{s}{2}R = -\frac{s}{2}\overset{\parallel}{R} + \frac{1}{2}(k^2 - K^2)$$

$$Ein_{\perp\parallel a} = Ric_{\perp\parallel a} = \overset{\parallel}{\nabla}_c K_a{}^c - \overset{\parallel}{\nabla}_a k$$

vnoření do Einsteinova prostoru

$$Ric - \frac{1}{2}R\,g + \Lambda g = 0 \qquad \frac{1}{L^2} = \frac{2\Lambda}{(m-1)(m-2)} \qquad Ric = \frac{1}{m}R\,g = \frac{m-1}{L^2}g \qquad R = \frac{m(m-1)}{L^2}$$

$$\Rightarrow\; R - 2s\,Ric_{\perp\perp} = \frac{m(m-1)}{L^2} - 2s^2\frac{m-1}{L^2} = \frac{n(n-1)}{L^2} \qquad n = m-1$$

$$\text{G-C} \Rightarrow\; \frac{1}{\ell^2} \equiv \frac{1}{n(n-1)}\overset{\parallel}{R} = \frac{1}{L^2} + \frac{s}{n(n-1)}(k^2 - K^2) \qquad (\text{nemusí konst})$$

vnoření do maximálně symetrického prostoru

$$R_{abcd} = \frac{1}{L^2}(g_{ac}g_{bd} - g_{ad}g_{bc}) \qquad Ric_{ab} = \frac{m-1}{L^2} g_{ab} \qquad R = \frac{m(m-1)}{L^2} = \text{konst}$$

$$\Downarrow$$

$$\overset{\shortparallel}{R}_{abcd} = \frac{1}{L^2}(q_{ac}q_{bd} - q_{ad}q_{bc}) + s(K_{ac}K_{bd} - K_{ad}K_{bc})$$

$$\overset{\shortparallel}{Ric}_{ab} = \frac{m-1}{L^2} q_{ab} + s(k K_{ab} - K^2_{ab})$$

$$\frac{1}{\ell^2} \equiv \frac{1}{m(m-1)} \overset{\shortparallel}{R} = \frac{1}{L^2} + \frac{s}{m(m-1)}(k^2 - K^2)$$

$$\overset{\shortparallel}{\nabla}_a K_{bc} = \overset{\shortparallel}{\nabla}_b K_{ac} \qquad \overset{\shortparallel}{\nabla}_c K_a{}^c = \overset{\shortparallel}{\nabla}_a k$$

umbilické vnoření

$$K_{ab} = \frac{1}{m} k\, q_{ab} \qquad K^2_{ab} = \frac{1}{m^2} k^2 q_{ab} \qquad K^2 = \frac{1}{m} k^2 \qquad m^2 \varkappa^2 = s k^2$$

$$\Downarrow$$

$$R_{\shortparallel a \shortparallel b \shortparallel c \shortparallel d} = \overset{\shortparallel}{R}_{abcd} - \varkappa^2 (q_{ac}q_{bd} - q_{ad}q_{bc})$$

$$Ric_{\shortparallel a \shortparallel b} - s R_{\perp \shortparallel a \perp \shortparallel b} = \overset{\shortparallel}{Ric}_{ab} - (m-1)\varkappa^2 q_{ab}$$

$$R - 2s\, Ric_{\perp\perp} \equiv -2s\, Ein_{\perp\perp} = \overset{\shortparallel}{R} - m(m-1)\varkappa^2$$

$$R_{\shortparallel a \shortparallel b \shortparallel c \perp} = \overset{\shortparallel}{\nabla}_a k\, q_{bc} - \overset{\shortparallel}{\nabla}_b k\, q_{ca}$$

$$Ric_{\perp \shortparallel a} = -\frac{m-1}{m} \overset{\shortparallel}{\nabla}_a k$$

$$\Downarrow$$

$$\frac{1}{\ell^2} \equiv \frac{1}{m(m-1)} \overset{\shortparallel}{R} = \frac{1}{m(m-1)}(R - 2s\, Ric_{\perp\perp}) + \varkappa^2$$

$$= -\frac{2s}{m(m-1)} Ein_{\perp\perp} + \varkappa^2$$

umbilické vnoření do maximálně sym. prostoru

$$\overset{\shortparallel}{R}_{abcd} = \left(\frac{1}{L^2} + \varkappa^2\right)(q_{ac}q_{bd} - q_{ad}q_{bc}) \qquad \frac{1}{\ell^2} = \frac{1}{L^2} + \varkappa^2$$

$$\overset{\shortparallel}{Ric}_{ab} = \frac{m-1}{\ell^2} q_{ab}$$

$$\overset{\shortparallel}{R} = \frac{m(m-1)}{\ell^2}$$

$$\overset{\shortparallel}{\nabla}_a k = 0$$

$$\frac{1}{\ell^2} = \frac{1}{L^2} + \varkappa^2 = \text{konst} \qquad \varkappa^2 = s\left(\frac{k}{m}\right)^2$$

Vnoření plochy do 3D max sym. pr

$m=3 \quad n=2 \quad s=+ \quad \operatorname{sign} g = (++)$

$$K = k_+ e^+e^+ + k_- e^-e^- \qquad g = e^+e^+ + e^-e^-$$

$$k = k_+ + k_- \qquad K^2 = k_+^2 + k_-^2 \qquad k^2 - K^2 = 2k_+k_-$$

vnoření do $\mathbb{E}^3$ $\quad R = 0 = \frac{1}{L^2}$

Gauss-Codazzi $\Rightarrow$

$$\frac{1}{\ell^2} \equiv \frac{1}{2}\mathcal{R} = k_+k_-$$

Theorema Egregium (Gauss)

lat. "pozoruhodný" teorém

vnější křivost

vnitřní křivost

vnoření do max. sym. pr = sféra/eukl./Lobač.

$$\frac{1}{\ell^2} \equiv \frac{1}{2}\mathcal{R} = \frac{1}{L^2} + k_+k_-$$

> 0 sféra S^3

$= 0$ euklidovský pr $\mathbb{E}^3$ $\qquad R = \frac{6}{L^2}$

< 0 Lobačevského pr $\mathbb{H}^3$